OCR
Oxford Cambridge and RSA

Wednesday 17 May 2017 – Morning

AS GCE MATHEMATICS

4721/01 Core Mathematics 1

QUESTION PAPER

5200862324

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4721/01
- List of Formulae (MF1)

Other materials required:
None

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are **not** permitted to use a calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

No calculator can be used for this paper

Answer **all** the questions.

1 Express $\frac{2+\sqrt{7}}{\sqrt{7}-2}$ in the form $a+b\sqrt{7}$, where a and b are rational numbers. **[3]**

2 Solve the simultaneous equations

$$y = x^2 - 6x, \qquad 2y + x - 6 = 0.$$ **[5]**

3 It is given that $f(x) = (3+x^2)(\sqrt{x} - 7x)$. Find $f'(x)$. **[5]**

4 Sketch the curve $y = -\frac{1}{2}(x+1)^2 + 2$, giving the coordinates of the turning point and indicating all points of intersection with the axes. **[5]**

5 Find the roots of the equation $4t^{\frac{2}{3}} = 15 - 17t^{\frac{1}{3}}$. **[5]**

6 **(i)** Express $3x^2 - 5x + 1$ in the form $a(x+b)^2 + c$. **[4]**

(ii) Work out the value of the discriminant of $3x^2 - 5x + 1$ and hence state the number of real roots of the equation $3x^2 - 5x + 1 = 0$. **[2]**

7 **(i)** Find the x values of the stationary points of the curve $y = 2x^4 - x^2$. **[3]**

(ii) Determine, in each case, whether the stationary point is a maximum point or a minimum point. **[2]**

(iii) Hence state the set of values of x for which curve $2x^4 - x^2$ is a decreasing function. **[2]**

8 **(i)** Sketch the curve $y = -2\sqrt{x}$. **[2]**

(ii) The curve $y = -2\sqrt{x}$ is translated by three units in the negative x direction. State the equation of the curve after it has been translated. **[2]**

(iii) Describe fully a single transformation that transforms the curve $y = -2\sqrt{x}$ to $y = -3\sqrt{5x}$. **[2]**

9 A curve has equation $y = 2x^2 + x - 10$.

(i) Determine the set of values of x for which the graph of the curve lies above the x-axis. **[4]**

(ii) The line $3x + y = c$ is a tangent to the curve. Find the value of c. **[5]**

10 The circle $x^2 + y^2 - 8x + 2y = 0$ passes through the origin O. Line OA is a diameter to this circle.

(i) Find the equation of the line OA, giving your answer in the form $ax + by = 0$, where a and b are integers. **[5]**

(ii) The tangent to the circle at point A meets the x-axis at the point B. Find the area of triangle OAB. **[6]**

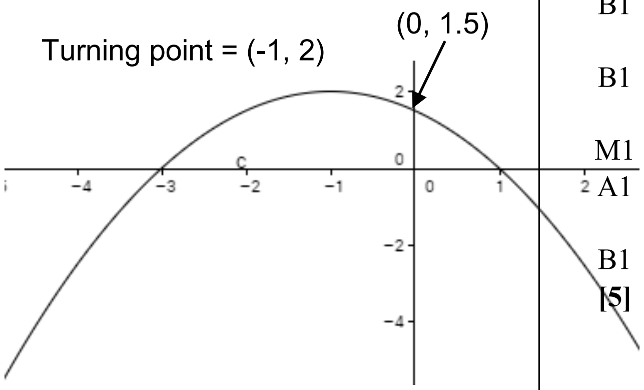
11 The normal to the curve $y = \frac{k}{x^2}$ at the point where $x = -3$ is parallel to the line $\frac{1}{2}y = 2 + 3x$.

(i) Determine the value of the constant k. **[6]**

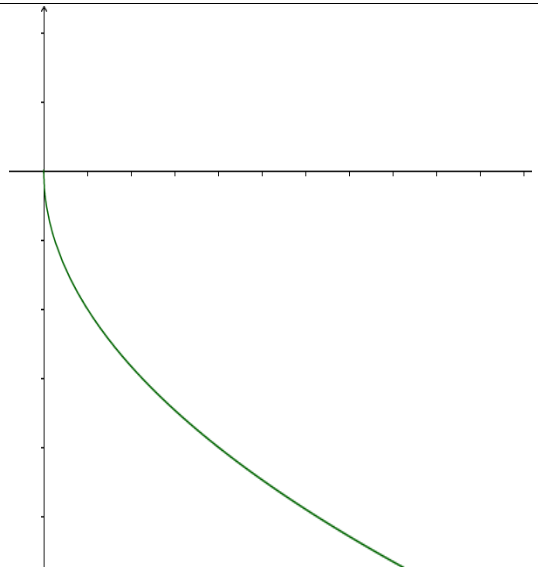
(ii) Find the equation of the normal where $x = -3$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[4]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1			$\frac{2+\sqrt{7}}{\sqrt{7}-2}\times\frac{\sqrt{7}+2}{\sqrt{7}+2}$	M1	Attempt to rationalise the denominator – must attempt to multiply. (May use $-\sqrt{7}-2$)	Alternative: **M1** Correct method to solve simultaneous equations formed from equating expression to $a+b\sqrt{7}$
			$\frac{11+4\sqrt{7}}{7-4}$	A1	Either numerator or denominator correct and simplified to no more than two terms	**A1** Either a or b correct
			$\frac{11}{3}+\frac{4\sqrt{7}}{3}$	A1 **[3]**	Fully correct and simplified. Allow $\frac{11+4\sqrt{7}}{3}$, terms in any order **Do not ISW** if then incorrect	**A1** Both correct Do not allow $\frac{-11-4\sqrt{7}}{-3}$ for last A1.
2			$2(x^2-6x)+x-6=0$	M1*	Substitute for x/y to eliminate one of the variables	If x eliminated: $y=(6-2y)^2-6(6-2y)$
			$2x^2-11x-6=0$	A1	Correct 2/3-term quadratic in solvable form	$4y^2-13y=0$
			$(2x+1)(x-6)=0$	M1* dep	Attempt to solve resulting quadratic. **See appendix 1.**	$y(4y-13)=0$
			$x=-\frac{1}{2}, x=6$	A1	x values correct	**Spotted solutions:** If **M0 DM0** **SC B1** One correct pair **www**
			$y=\frac{13}{4}, y=0$	A1 **[5]**	y values correct Award **A1 A0** for one pair **correctly found from correctly factorised quadratic**	**SC B1** Second correct pair **www** Must show on both line and curve (Can then get 5/5 if both found **www** and exactly two solutions justified)
3			$\sqrt{x}=x^{\frac{1}{2}}$ seen or implied	B1		Alternative using product rule: **B1** as main scheme
			$3x^{\frac{1}{2}}-21x+x^{\frac{5}{2}}-7x^3$	M1 A1	Attempts to expand brackets with 3/4 terms soi Correct expression for $f(x)$ in index form	**M1*** Clear attempt at $uv'+vu'$ **A1** All terms fully correct
			$\frac{3}{2}x^{-\frac{1}{2}}-21+\frac{5}{2}x^{\frac{3}{2}}-21x^2$	M1 A1 **[5]**	Attempt to differentiate their expression with at least one non-zero term correct Correct expression for $f'(x)$ **cao ISW** any attempts to put back into root form.	**M1*dep** Attempt to expand brackets with at least two terms simplified correctly **A1** Correct expression for $f'(x)$

Question	Answer	Marks	Guidance	
4	 	B1 B1 M1 A1 B1 [5]	Negative parabola Turning point at $(-1, 2)$; coordinates must be labelled on graph or clearly stated elsewhere Correct method to find roots* Correct x intercepts $(1,0)$ and $(-3, 0)$ Correct y intercept $(0, \frac{3}{2})$ **NB** – Do not award 5/5 if sketch inconsistent with stated values e.g. turning point shown in wrong quadrant etc. Withhold one B1.	For **first mark** must clearly be a parabola – must not stop at or before x axis, do not allow straight line sections drawn with a ruler or tending to extra turning points etc. Must not be a finite plot. * If not using given form to solve, M mark only available for attempt to solve $k\left(-\frac{1}{2}x^2 - x + \frac{3}{2}\right) = 0$. **See appendix 1.**
5	$k = t^{\frac{1}{3}}$ $4k^2 + 17k - 15 = 0$ $(4k-3)(k+5) = 0$ $k = \frac{3}{4}, k = -5$ $t = \frac{27}{64}, t = -125$	M1* M1* dep A1 M1 A1 [5]	Substitute for $t^{\frac{1}{3}}$ to obtain a quadratic expression Rearrange and attempt to solve resulting quadratic equation. **See appendix 1.** Correct values of k Attempt to cube at least one value Final answers correct	Alternative**: M2** Rearrange and factorise into two brackets containing $t^{\frac{1}{3}}$. **See appendix 1.** **SC** If straight to formula with no evidence of substitution at start and no cubing/cube rooting at end, then **B1** for $\frac{-17\pm\sqrt{(17^2-4\times4\times-15)}}{2\times4}$ or better **No marks** if whole equation cubed etc. **Spotted solutions:** If **M0 DM0** or **M1 DM0** **SC B1** $t = \frac{27}{64}$ **www** **SC B1** $t = -125$ **www** (Can then get 5/5 if both found **www** and exactly two solutions justified)

Question		Answer	Marks	Guidance	
6	(i)	$3(x^2-\frac{5}{3}x)+1$ $3[(x-\frac{5}{6})^2-\frac{25}{36}]+1$ $3(x-\frac{5}{6})^2-\frac{13}{12}$	B1 B1 M1 A1 **[4]**	$a=3$ $b=-\frac{5}{6}$ (not $\frac{-\frac{5}{3}}{2}, \frac{-2.5}{3}$) $1-3b^2$ or $3\times(\frac{1}{3}-b^2)$ $c=-\frac{13}{12}$. Allow $-\frac{39}{36}$ etc.	$3(x-\frac{5}{6})^2+\frac{13}{12}$ **B1 B1 M0 A0** $3(x-\frac{5}{6})\ -\frac{13}{12}$ **4/4 BOD** $3(x-\frac{5}{6}x)^2-\frac{13}{12}$ **B1 B0 M1 A0** $3(x^2-\frac{5}{6})^2-\frac{13}{12}$ **B1 B0 M1 A0** $3x(x-\frac{5}{6})^2-\frac{13}{12}$ **B0 B1 M1 A0** $3(x^2-\frac{5}{6})-\frac{13}{12}$ **B1 B0 M1 A0** $3(x+\frac{5}{6})^2-\frac{13}{12}$ **B1 B0 M1 A0**
	(ii)	$(-5)^2-4.\ 3.1=13$ So 2 real roots	B1 B1ft **[2]**	ft their discriminant e.g. "– 25 – 12 = – 37 so no roots" scores **B0 B1ft**	Use of $\sqrt{b^2-4ac}$ can score **B0 B1**
7	(i)	$\frac{dy}{dx}=8x^3-2x$ At stationary points $8x^3-2x=0$ $x=\frac{1}{2}, x=-\frac{1}{2}, x=0$	B1 M1 A1 **[3]**	Correct differentiation Sets their derivative to zero Correctly obtains **all three** roots.	**B0 M0** if expression is integrated and equated to zero. Do not accept $\pm\sqrt{\frac{1}{4}}$.
	(ii)	$\frac{d^2y}{dx^2}=24x^2-2$ When $x=\pm\frac{1}{2}$, $\frac{d^2y}{dx^2}>0$ so minimum, maximum when $x=0$	M1 A1 **[2]**	Uses correct method to find nature of **at least one** stationary point e.g. substitution into second derivative (at least one term correct from their first derivative in (i)) and consider sign. Correct conclusions for all three points **www**	**Alternate valid methods include:** 1) Determining sign of gradient at either side of stationary point 2) Evaluating y at, and either side of, stationary point 3) Correct sketch Working must be fully correct to obtain the **A** mark

Question		Answer	Marks	Guidance	
	(iii)	$x < -\frac{1}{2}$, $0 < x < \frac{1}{2}$	B2 **[2]**	Both regions correct (allow B1 for one correct region)	Condone use of ≤ instead of <. Condone e.g. $\sqrt{\frac{1}{4}}$ here.
8	**(i)**		B1 B1 **[2]**	Correct shape in correct quadrant – must intend to go through (0, 0) Sketch must also : • Start at (0,0) • Have fully correct curvature – does not tend to a horizontal asymptote • Not be a finite "plot"	
8	**(ii)**	$y = -2\sqrt{x+3}$	M1 A1 **[2]**	Translates curve by +/— 3 parallel to the *x*-axis Fully correct, must have "*y* ="	
	(iii)	Stretch Scale factor $\frac{3\sqrt{5}}{2}$ parallel to the *y*-axis (Scale factor $\frac{4}{45}$ **oe** parallel to the *x*-axis)	B1 B1 **[2]**	**Must** use stretch/stretched/stretching… Allow "factor" or "SF" for "scale factor" For "parallel to the *y* axis" allow "vertically", "in the *y* direction". Do not accept "in/on/across/up the *y* axis" , "SF 5 units" Apply the same principles to alternative correct answer:	Allow **first B1 only** for multiple transformations **provided all are stretches**. Allow $\frac{\sqrt{45}}{2}$, $\sqrt{\frac{45}{4}}$ etc. for $\frac{3\sqrt{5}}{2}$ **B0B1** is possible e.g. "Enlarge by scale factor…" etc. but not for (e.g.) "translate by scale factor…" or similar.

Question			Answer	Marks	Guidance	
9	(i)		$(2x+5)(x-2)=0$	M1	Correct method to find roots. **See appendix 1.**	
			$-\frac{5}{2}, 2$	A1	Roots correct	
			$x<-\frac{5}{2}, x>2$	M1	Chooses the "outside region" for their roots	**NB** e.g. $-\frac{5}{2}>x>2$ scores **M1A0** if correct answer not previously seen. Must be strict inequalities for **A** mark
				A1	Allow "$x<-\frac{5}{2}, x>2$", "$x<-\frac{5}{2}$ **or** $x>2$" but do not allow "$x<-\frac{5}{2}$ **and** $x>2$"	
				[4]		
9	(ii)		Gradient of line = – 3	B1	Stated or used.	Look out for using 3 instead of –3. This gives $x=\frac{1}{2}$ which also leads to y = – 9. **B0B1M1A0A0** Max 2/5
			$\frac{dy}{dx}=4x+1$	B1	Correct differentiation	
			$4x+1=-3$	M1	Equates their derivative with their gradient of line	
			$x=-1$	A1	x correct	
			$y=-9$ $-9=-3(-1)+c \Rightarrow c=-12$	A1	c correct. Could also obtain from substituting $x=-1$ into $2x^2+x-10=c-3x$.	
			OR	**OR**		
			$2x^2+x-10=c-3x$	M1	Equates line and curve	
			$2x^2+4x-10-c=0$	A1	Obtains correct quadratic = 0	
			Tangent $\Rightarrow b^2-4ac=0$	M1	Uses tangency implies $b^2-4ac=0$	
			$\Rightarrow 4^2-4.2.(-10-c)=0$	A1	Fully correct substitution	
			$c=-12$	A1	c correct	
				[5]		

Question		Answer	Marks	Guidance	
10	(i)	$(x-4)^2 - 16 + (y+1)^2 - 1 = 0$ $(x-4)^2 + (y+1)^2 = 17$	M1	Correct method to find centre of circle	e.g. $(x \pm 4)^2$ and $(y \pm 1)^2$ seen (or implied by correct answer)
		Centre = (4, – 1)	A1	Correct centre soi.	**M** can be implied by correct centre.
		$m = -\frac{1}{4}$	B1	Gradient of OA correct (could use OC or CA) [A = (8, – 2) is not required for this part, but may be used]	Note: Centre (– 4, 1) leads to "correct" answer. **M1A0B0M1A0** Max 2/5
		$y = -\frac{1}{4}x$	M1	Attempts equation of straight line through O or A or centre of the circle with their calculated gradient.	
		$x + 4y = 0$	A1 **[5]**	**www** Correct equation in required form i.e. $k(x+4y) = 0$ for integer k, allow $0 = 4y + x$ etc.	<u>Alternative for first three marks:</u> **M1** Attempt at implicit differentiation as evidenced by $2y\frac{dy}{dx}$ term **A1** $2x + 2y\frac{dy}{dx} - 8 + 2\frac{dy}{dx} = 0$ and substitutes O to obtain $\frac{dy}{dx} = 4$ **B1** Find correct negative reciprocal
10	(ii)	A = (8, – 2) $m' = 4$	B1ft B1ft	Must be seen/used in (ii); ft their centre ft their gradient in (i)	
		$y + 2 = 4(x - 8)$	M1	Attempts equation of perpendicular line through their A. (**Not** (4, –1).)	If centre used here, max **B1B1**, 2/6.
		When $y = 0, x = \frac{17}{2}$	M1	Attempt to find x value of point B from their equation of perpendicular line	Equation of line/B may not be seen explicitly.
		Area = $\frac{1}{2} \times \frac{17}{2} \times 2 = \frac{17}{2}$	M1	Attempt to find area of OAB e.g. $\frac{1}{2} \times$ their OB $\times$ their 2, or $\frac{1}{2} \times$ their OA$\times$ their AB, or split into two triangles	Must have used a valid method to find B. OA = $\sqrt{68}$, AB = $\sqrt{\frac{17}{4}}$
			A1 **[6]**	Accept 8.5 or equivalent fractions but not unsimplified surds. **www**	Look out for "correct" answer from wrong coordinates – **A0**.

Question			Answer	Marks	Guidance	
11	(i)		Gradient of given line = 6	B1	soi as gradient of the line	Can be implied by use of $-\frac{1}{6}$
			Perpendicular gradient = $-\frac{1}{6}$	M1	Uses product of perpendicular gradients is – 1 at some point; may be implied by later working.	
			$\frac{dy}{dx}=-2kx^{-3}$	M1 A1	Attempt to differentiate (ax^{-3} seen) Fully correct	
			$-\frac{1}{6}=-2k(-3)^{-3}$	M1	Equates their derivative at $x=-3$ with their perpendicular gradient	e.g. $-\frac{27}{2k}=6$ (implies first M1)
			$k=-\frac{9}{4}$	A1 **[6]**	Correct value of k. Allow $-\frac{27}{12}$ etc.	
	(ii)		When $x=-3$, $y=-\frac{9}{4(-3)^2}=-\frac{1}{4}$	B1	Correct value of y **www**	
			$y+\frac{1}{4}=6(x+3)$	M1	Attempts equation of straight line through $(-3, y)$, any non-zero gradient. y must be from their k but allow slips for M mark.	
			$24x-4y+71=0$	A1ft	Correct equation in any form – gradient 6 but ft their value of $\frac{k}{9}$. Allow 6 (x – –3)	For the first A mark, allow follow through their value of k – straight line through (–3, their $\frac{k}{9}$) with correct gradient of 6 e.g. k = 81 leads to $y-9=6(x+3)$
				A1 **[4]**	Correct equation in required form i.e. $a(24x-4y+71)=0$ for integer a, terms in any order. **cao**	